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On the critical behaviour of superfluid Fermi liquids

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Abstract. The influence of Fermi liquid Landau-type interactions on the critical behaviour of superconductors and superfluids in the presence of DC magnetic fields and superflows is studied. Paramagnetic and diamagnetic instability phenomena are described in detail for singlet s- and triplet p-paired superconducting Fermi liquids. The impact of Landau interactions on both instability effects is found to be significant. Inhomogeneous Larkin–Ovchinnikov superfluid states are also considered in the case of strongly interacting systems. The influence of Landau interactions on critical depairing superflows for s- and p-paired superfluids is determined. Applications of the results obtained to the description of superconductors, superfluid ^3He and ^3He–^4He mixtures as well as the rare earth ternary compound $ErRh_4B_4$ are discussed.

1. Introduction

Recent experimental developments with new superconducting materials, such as the heavy-fermion superconductors, rare earth ternary compounds and high-T_c superconductors, have increased the interest in theoretical studies of various modifications of the simple BCS model. One important possibility is the inclusion of interactions in the particle–hole channel leading to Fermi-liquid-type corrections. These corrections do not manifest themselves as renormalisation constants only, but despite common opinion, they essentially influence the ground state of the system, as will be demonstrated in this paper. In the presence of external fields (of the order of the energy gap) the equilibrium state of a superfluid Fermi liquid (SFL) appears to be highly sensitive to the magnitude of the appropriate part of the quasi-particle Landau-type interaction. This field can be considered as a DC magnetic field that acts on the particles via both Pauli and orbital (for charged systems) terms in a free-particle Hamiltonian. The relative smallness of this field in comparison to the Fermi energy is, however, necessary when we neglect the field dependence of the Fermi liquid and pairing interactions (the neglect of these corrections is consistent with the accuracy of the weak coupling theory of superconductors).

With regard to the DC magnetic field, both its paramagnetic and diamagnetic actions on quasi-particles are 'dressed' with appropriate moments of Landau interaction which influence the critical behaviour of the superconducting state. For neutral singlet s- and triplet p-type superfluid Fermi liquids (s-SFLs, p-SFLs), the so-called paramagnetic instability phenomenon (Maki and Tsuneto 1964, Sarma 1963) is highly sensitive to the spin-exchange Landau interaction. For the s-SFL state and $b_0 > 1.195$ (b_0 is the isotropic part of the spin-exchange quasi-particle interaction) the paramagnetic instability phenomenon is completely damaged and the superfluid–normal phase transition is of second order on the whole critical curve. For $-1 < b_0 < 1$ the usual Maki–Tsuneto–Sarma critical behaviour is observed with a single tricritical point, while for $1 < b_0 < 1.195$ two tricritical points exist. A similar property also holds for p-SFL, although a more complicated order parameter structure manifests itself in energy competition between a variety of possible superfluid states. The relevant considerations are presented in §§ 1 and 2.

With regard to superfluid ^{3}He-B, for which the weak coupling approach to a p-paired condensate is applicable, analyses of the magnetic properties of ^{3}He-B have been made, e.g., Jacak (1981), Gonczarek and Jacak (1978), Hasegawa (1980), Tewordt and Schopohl (1979), Schopohl (1982) and McInrney (1980, 1981). Numerical solutions have been found for Landau parameters specific to ^{3}He (Schopohl 1982) and therefore they do not reveal the typical p-paired Fermi liquid polycritical behaviour associated with other interaction parameter values.

In order to state whether the determined polycritical points are real tricritical points or Lifshitz points, inhomogeneous states are also considered. The stability of the so-called Larkin–Ovchinnikov (LO) state (Larkin and Ovchinnikov 1964, Fulde and Ferrell 1964) also turns out to be strongly affected by the spin-exchange Landau interaction. For s-SFL it is established in § 3 that for $-1 < b_0 < -0.122$ the LO state does not minimise the free energy at any DC magnetic field value, while for $-0.122 < b_0 < 3.40$, at certain magnetic field values, a first-order phase transition BCS–LO occurs (for $b_0 > 3.40$ it is a second-order phase transition). The positive value of b_0 also enhances the energetical distances between stable LO and unstable BCS or normal states. Recall that for an s-paired Fermi gas the appropriate energetical distances are very small and can be easily cancelled by spin–orbit impurity scattering (Takada 1970, Aslamazov 1969). The analysis in § 1–3, although complete for a neutral SFL, cannot generally be applied to the description of real electronic superconductors (note, however, that p-SFL is a good model for superfluid ^{3}He) because the orbital depairing action of the magnetic field usually greatly exceeds the similar action of the Pauli term. Thus diamagnetic effects play a dominant role in real electronic materials with regard to their superfluid properties. The so-called Pauli-limiting field, which is used in the evaluation of the upper critical field for type II superconductors (Alexander *et al* 1985), should be considered as an approximation only. Note, however, that for Kivelson's interpretation of the Anderson resonating valence band state (with regard to high-T_c superconductors), the uncharged fermionic solitons can condense into neutral SFLs for which paramagnetic effects would be important.

Some superconducting materials have magnetic admixtures, in which the paramagnetic depairing mechanism is important. The reason for this is the enlargement of the effective Bohr constant due to the ferromagnetic alignment of admixture spins, as in the system considered by Fujita *et al* (1984). This system, known as a magnetically polarisable superconductor, can be used as a model to explain the critical behaviour in the rare earth ternary compound $ErRh_4B_4$. However, due to the interplay of magnetism and superfluidity in $ErRh_4B_4$, some modification of the paramagnetic theory is needed. In § 6 we evaluate b_0 in order to fit the position of the cross-over point for $ErRh_4B_4$ according to the experimental results of Grabtree *et al* (1982).

On the other hand, the description of the paramagnetic critical behaviour of the singlet-paired superfluid system forms an appropriate model for the magnetic properties of superfluid ^{3}He in the mixture ^{3}He–^{4}He where the singlet-type pairing interaction seems to be favourable due to the bosonic medium of ^{4}He atoms. Unfortunately, experimental evidence for this superfluidity has not been yet obtained, probably because of the very low transition temperature of 10^{-4} K (Ivanova and Mejerovich 1986).

In the case of the usual charged superconductors the situation is different. Since the canonical momentum operator is non-local (containing the magnetic field vector potential), the inclusion of the orbital magnetic term is inconvenient (in momentum representation). Nevertheless, the essential simplification is justified in the London limit when the DC magnetic field penetration depth considerably exceeds the coherence length of the superconducting state. This simplification consists in the assumption that the superfluid velocity (the gauge-invariant combination of the vector potential and gradient of order parameter phase) is spatially uniform. Moreover, due to the uniqueness of the distance scale, governed in the London limit by the coherence length, one can consider the superfluid velocity to be of the order of $\xi_0 h$, where ξ_0 is the coherence length in interatomic distance units, and $h = \mu_B H$, where H is the magnetic field. Usually in metals $\xi_0 \simeq 10^2$–10^4, so that the Pauli term (of the order of h) appears to be completely negligible (although for high-T_c Cu oxides ξ_0 is considerably less and the Pauli term seems to be much more important). Thus the depairing action of the orbital term (treated even in the framework of the London limit) is crucial with regard to the magnetic properties of ordinary superconducting metals. For the case of a superfluid Fermi liquid when the electronic interaction is taken into account, one important observation is that the superfluid velocity v_s (as well as the magnetic field potential) is 'dressed' with an interaction via the vector part of a spinless Landau function (as is the effective mass renormalisation). This is due to the vectorial nature of v_s (note that the magnetic field entering via the Pauli term is 'dressed' with a spin exchange interaction as the pseudovector). The diamagnetic critical effects for both s-SFL and p-SFL are considered in § 5. The diamagnetic instability phenomenon is described by analogy with the paramagnetic one. Note, however, that the polycritical behaviour is completely destroyed by quasi-particle interaction with $a_1 > -\frac{1}{7}$ ($m^* = m(1 + a_1)$ is the effective mass formula for the isotropic case). This explains why the diamagnetic instability phenomenon was not known for a simple BCS gas.

Finally, we emphasise that considerations of the orbital action of a DC magnetic field in the framework of the local (i.e., London) limit also provide an adequate model of the states of a neutral SFL with superflows, which in the case of p-SFL are applicable to the description of ^{3}He (of its B phase at least, since for the A phase the strong coupling correction needs to be taken into account; see Vollhardt *et al* 1980). We have found that the critical superflow is affected by the Landau interaction, in contrast with earlier opinion. This influence manifests itself since the first-order phase transitions to normal phase do not allow the superflow to attain its maximal value. This important property occurs for some regions of a_1 values, for both s- and p-SFLs.

2. Paramagnetic critical properties of neutral s-SFLs

Let us consider a neutral SFL with singlet s-type pairing (s-SFL) in the presence of a DC magnetic field of the order of T_c/μ_B. In the framework of the weak coupling approach one can find self-consistent equations for the anomalous and normal parts of the mass operator, which coincide with the gap parameter and renormalisation of the paramagnetic term (the Pauli term in the Hamiltonian), respectively. The explicit forms of the equations are as follows (cf. Jacak and Krzyżanowski 1985):

$$\Delta = \tfrac{1}{2}\Delta\lambda \int \frac{d\Omega}{4\pi} \int_0^{\omega_D} d\xi \frac{1}{E}\left(\tanh\frac{E+H}{2T} + \tanh\frac{E-H}{2T}\right) \tag{1}$$

$$\boldsymbol{\Sigma}(\hat{\boldsymbol{p}}) = -\tfrac{1}{2} \sum_{l=0,2,\ldots} b_l(2l+1) \int \frac{\mathrm{d}\Omega'}{4\pi} P_l(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}}')\hat{H} \times \int_0^\infty \mathrm{d}\xi \left(\tanh\frac{E+H}{2T} - \tanh\frac{E-H}{2T}\right) \tag{2}$$

where $\lambda = N_0 g_0$ is the pairing constant, $b_l = N_0 f_l^a$ are the dimensionless spin-exchange Landau parameters, and N_0 is the density of states at the Fermi surface. We consider the isotropic system, so that in equation (2) the expansion of the Landau function in Legendre polynomials has been applied:

$$f^{s(a)}(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}}') = \sum_l \frac{1}{2l+1} f_l^{s(a)} P_l(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}}').$$

The renormalised magnetic field $\boldsymbol{H} = \boldsymbol{h} + \boldsymbol{\Sigma}$. It can be seen that for the paramagnetic spin magnetisation $\boldsymbol{M}$ the following holds:

$$N_0 \int \frac{\mathrm{d}\Omega}{4\pi} \boldsymbol{\Sigma}(\hat{\boldsymbol{p}}) = -b_0 \boldsymbol{M}. \tag{3}$$

The above system of equations is unmanageable at arbitrary temperatures, although at $T = 0$ it can be solved analytically. Let us first consider $T = 0$ limit, in which case equations (1) and (2) have the following form (provided the hypothetical possibility of a spontaneous breakdown of rotational symmetry is not considered):

$$\Delta = \Delta\lambda \int_{\Theta(H-\Delta)(H^2-\Delta^2)^{1/2}}^{\omega_\mathrm{D}} \mathrm{d}\xi\,(\xi^2 + \Delta^2)^{-1/2} \tag{4}$$

$$\Sigma = -b_0 \Theta(H - \Delta)(H^2 - \Delta^2)^{1/2} \tag{5}$$

where Θ is the Heaviside step function. Equations (4) and (5) can be readily solved and the explicit expressions for Δ and Σ are

$$\Delta = \Delta_0 \begin{cases} 1 & \text{for } h \leqslant \Delta_0 \\ \left(\dfrac{1 + b_0 - 2h/\Delta_0}{b_0 - 1}\right)^{1/2} & \text{for } h_1 \leqslant h \leqslant h_2 \\ 0 & \text{for } h \geqslant h_2 \end{cases} \tag{6}$$

$$\boldsymbol{\Sigma} = -b_0 \hat{h} \begin{cases} 0 & \text{for } h \leqslant \Delta_0 \\ (h - \Delta_0)/(b_0 - 1) & \text{for } h_1 \leqslant h \leqslant h_2 \\ h/(1 + b_0) & \text{for } h \geqslant h_2 \end{cases} \tag{7}$$

where

$$h_1 = \min(\Delta_0, \Delta_0(1 + b_0)/2) \qquad h_2 = \max(\Delta_0, \Delta_0(1 + b_0)/2). \tag{8}$$

Here $\Delta_0 = \Delta(T = 0, h = 0) = \pi T_\mathrm{c}/\mathrm{e}^c$ and $c \simeq 0.577$ is the Euler constant.

From equations (6) and (7) it follows that the condition $b_0 = 1$ determines the irregular point of the system (4) and (5) where the topology of its solution changes. That is, for $b_0 > 1$ there exist unique non-zero solutions, while for $-1 < b_0 < 1$ two branches of non-zero solutions are available. Thus the point given by the conditions $T = 0$, $b_0 = 1$, $h = \Delta_0$ is the bifurcation point of the system of non-linear equations (4) and (5). From the physical point of view this point coincides with the tricritical point (at $T = 0$) at which the superfluid–normal phase transition changes its order. For $b_0 > 1$ it is a second-order phase transition and for $-1 < b_0 < 1$ first-order. The appropriate critical field (for the first-order phase transition) can be found by evaluating the field at which the energetical distance between superfluid and normal phase vanishes (similar to the first-order phase transition in a van der Waals gas). The difference in free energies of normal and superfluid phases can be found using the general Feynman method (i.e., integration with respect to interaction constants). The appropriate formula is

$$F_{\mathrm{S},h} - F_{\mathrm{N},h} = \int_0^{\Delta} \mathrm{d}\Delta' \left[\Delta' - \tfrac{1}{2}\lambda\Delta' \int_0^{\omega_{\mathrm{D}}} \mathrm{d}\xi \frac{1}{E}\left(\tanh\frac{E'+H}{2T} + \tanh\frac{E'-H}{2T}\right)\right]$$

$$+ \nu \int_{\Sigma_N}^{\Sigma} \mathrm{d}\Sigma' \left[\Sigma' + \frac{b_0}{2}\int_0^{\infty} \mathrm{d}\xi \left(\tanh\frac{\xi+H'}{2T} - \tanh\frac{\xi-H'}{2T}\right)\right] \tag{9}$$

where $F_{\mathrm{S},h}$ ($F_{\mathrm{N},h}$) is the free energy of the SFL (NFL is a normal FL) in the presence of a magnetic field: $E' = (\xi^2 + \Delta'^2)^{1/2}$, $H' = h + \Sigma'$, $\nu = -\lambda/b_0$ and $\Sigma_{\mathrm{N}} = -b_0 h/(1+b_0)$.

In the limit $T = 0$, equation (9) assumes the simple form:

$$F_{\mathrm{S},h} - F_{\mathrm{N},h} = \tfrac{1}{2}\lambda\{-\Delta^2/2 + H^2 + \Theta(H-\Delta)(H^2-\Delta^2)^{1/2} - [b_0/(1+b_0)] \times [h + \Sigma(1+b_0)/b_0]^2\}. \tag{10}$$

Taking into account equations (6), (7) and (10) we find that for $-1 < b_0 < 1$ the stable solution is the step function

$$\Delta = \Delta_0, \Sigma = 0 \qquad \text{for } 0 \leqslant h < \Delta_0[(1+b_0)/2]^{1/2}$$

$$\Delta = 0, \Sigma = \Sigma_{\mathrm{N}} \qquad \text{for } h > \Delta_0[(1+b_0)/2]^{1/2}.$$

At $h = \Delta_0[(1+b_0)/2]^{1/2}$ the first-order superfluid–normal phase transition is realised ($T = 0$). For $b_0 > 1$ the solution (6) and (7) is stable within the whole region of determination and the phase transition to the normal state is of the second order.

For non-zero temperatures only a numerical analysis is available. For several temperatures the solutions to equation (1) are plotted in figure 1. For $b_0 = 0$ they correspond to $\Delta(h)$, since for $b_0 = 0$, $H = h$. For $b_0 \neq 0$, however, equation (2) appears to be important, leading to a considerable change in the typical behaviour of a BCS gas described by Sarma (1963) and Maki and Tsuneto (1964) known as a paramagnetic instability phenomenon. They have determined the position of the tricritical point for a BCS gas, i.e., for $b_0 = 0$: $T^* = 0.556T_{\mathrm{c}}$ and $h^* = 0.540\Delta_0$.

For the BCS Fermi liquid the position of the tricritical point is highly sensitive to variations in b_0. It is convenient to account for this impact by determining the bifurcation points of the system (1) and (2). The necessity condition for irregular points of the system (1)–(2) leads to the following equation:

$$\left\{\int_0^{\omega_{\mathrm{D}}} \mathrm{d}\xi \left[\frac{1}{E^3}(\tanh A_+ + \tanh A_-) - \frac{1}{2TE^2}\left(\frac{1}{\cosh^2 A_+} + \frac{1}{\cosh^2 A_-}\right)\right]\right\}$$

$$\times \left[1 + \frac{b_0}{2}\int_0^{\infty} \mathrm{d}\xi \frac{1}{2T}\left(\frac{1}{\cosh^2 A_+} + \frac{1}{\cosh^2 A_-}\right)\right]$$

$$+ \frac{b_0}{2}\left[\int_0^{\infty} \mathrm{d}\xi \frac{1}{2TE}\left(\frac{1}{\cosh^2 A_+} - \frac{1}{\cosh^2 A_-}\right)\right]^2 = 0 \tag{11}$$

where $A_{\pm} = (E \pm H)/2T$. Taking this equation in the limit $\Delta \to 0$ one can determine

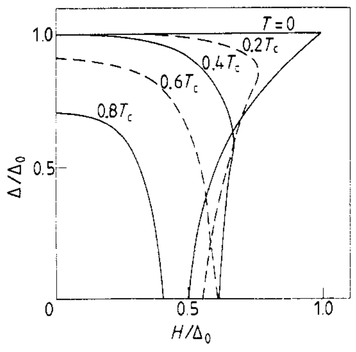

Figure 1. Solutions to the gap equation (1) $\Delta = \Delta(H)$ at various temperatures.

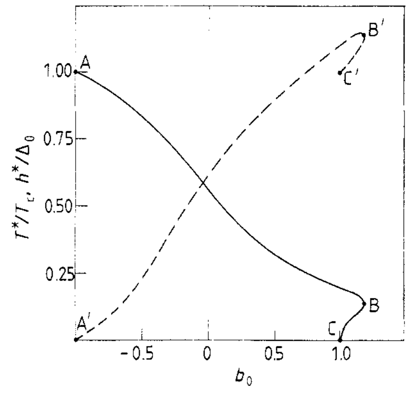

Figure 2. Coordinates of the tricritical point for an s-type superfluid Fermi liquid: T^* (full curve) and h^* (broken curve) versus b_0. Corresponding branches of both curves are denoted by the same letters.

the bifurcation points corresponding to the tricritical points on the phase transition curve. As a result the coordinates of the tricritical points versus b_0 are plotted in figure 2. Three types of behaviour are possible. For $-1 < b_0 < 1$ there is only one tricritical point with coordinates $T^*(b_0)$ and $h^*(b_0)$ in which the transition to normal phase changes its order: first order for $0 \leqslant T < T^*$, and second order for $T^* < T \leqslant T_c$. For $1 \leqslant b_0 < 1.195$ there are two tricritical points with coordinates $T_1^*(b_0), h_1^*(b_0)$ and $T_2^*(b_0), h_2^*(b_0)$ (see figure 2). The phase transitions are second order for $0 \leqslant T < T_1^*$ and $T_2^* < T \leqslant T_c$ ($T_1^* < T_2^*$) and first order for $T_1^* < T < T_2^*$. For $b_0 > 1.195$ no poly-critical point appears and the phase transition is second order for all $0 \leqslant T \leqslant T_c$. Hence in the region $b_0 > 1.195$ the quasi-particle Landau-type interaction completely destroys the paramagnetic instability phenomenon.

Phase diagrams typical for the above three cases ($b_0 = 0.30$, 1.05 and 1.20) are presented in figure 3. The appropriate critical curves (full lines) corresponding to first-order phase transitions are determined by evaluating the free energy according to equation (9). The broken and dotted lines correspond to 'superheating' and 'super-cooling' fields, respectively.

3. Paramagnetic critical properties of p-SFL

For triplet p-type pairing we deal with a much more complicated structure of the superconducting phase order parameter than that for s-SFL. The appropriate 3×3 complex matrix order parameter can be represented by suitably chosen spin tensors corresonding to appropriate irreducible representations of the rotational group. For the case where an external DC magnetic field is present this representation is especially convenient since the axial rotational symmetry along the field axis is maintained. Thus the Cooper pair states can be classified by means of the quantum numbers m; for the

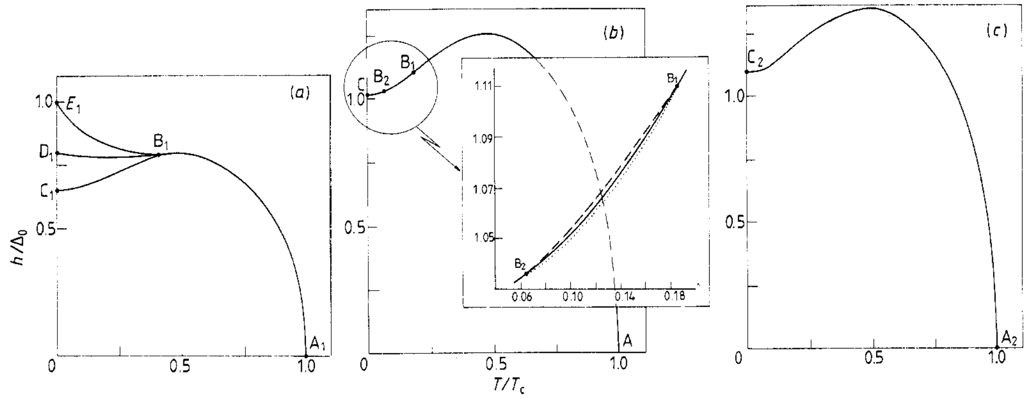

Figure 3. Phase diagrams T versus h for (a) $b_0 = 0.30$, (b) $b_0 = 1.05$ and (c) $b_0 = 1.20$.

p-triplet states we have $m = 0, 1, 2$. The three-dimensional $m = 0$ case is most important because of its relation to the order parameter of superfluid ^{3}He-B, which can also be described in the framework of weak coupling theory. If we denote by $\hat{q}$ the versor of the magnetic field, then the $m = 0$ type state can be represented as

$$\Delta = (d_{rs}\hat{p}_r\sigma_s)(\mathrm{i}\sigma_y)$$

$$d_{rs} = \Delta_1 \cos\theta(\delta_{rs} - \hat{q}_r\hat{q}_s) + \Delta_2\hat{q}_r\hat{q}_s + \Delta_1 \sin\theta\varepsilon_{rsl}\hat{q}_l \tag{12}$$

where Δ_1 and Δ_2 are longitudinal and transverse gap amplitudes, respectively, and the angle Θ describes the orientation of the spin and momentum spaces. Taking into account representation (12) one can write the self-consistent equations for the gap and renormalised magnetic field (cf. Schopohl 1982); here we have restricted ourselves to isotropic spin-exchange quasi-particle interaction:

$$\Delta_1 = \tfrac{3}{8}\lambda\Delta_1 \int_{-1}^{1} \mathrm{d}x\,(1 - x^2) \int_0^{\omega} \mathrm{d}\xi \left(\frac{\tanh(E_+/2T)}{E_+} + \frac{\tanh(E_-/2T)}{E_-}\right) \tag{13}$$

$$\Delta_2 = \tfrac{3}{4}\lambda\Delta_2 \int_{-1}^{1} \mathrm{d}x\,x^2 \int_0^{\omega} \frac{\mathrm{d}\xi}{E} \left(M_+ \tanh\frac{E_+}{2T} + M_- \tanh\frac{E_-}{2T}\right) \tag{14}$$

$$\Sigma = \tfrac{1}{4}b_0 \int_{-1}^{1} \mathrm{d}x \int_0^{\infty} \mathrm{d}\xi \left(-M_+ \tanh\frac{E_+}{2T} + M_- \tanh\frac{E_-}{2T}\right) \tag{15}$$

where

$$E = (\xi^2 + \Delta_2^2x^2)^{1/2} \qquad E_\pm = [(E \pm H)^2 + \Delta_1^2(1 - x^2)]^{1/2}.$$

Also $M_\pm = (E \pm H)E_\pm^{-1}$ and $\lambda = N_0g_1$ is the pairing constant.

According to the structure of these equations, three types of solutions are possible: the 1D axial state ($\Delta_1 = 0$, $\Delta_2 \neq 0$), the 2D planar state ($\Delta_2 = 0$, $\Delta_1 \neq 0$) and the BW anisotropic state (Balian and Werthamer 1963; with $\Delta_1 \neq 0$, $\Delta_2 \neq 0$). While the 1D state is completely energetically unfavourable, the competition between BW anisotropic and 2D states leads to a stabilisation of the BW state for $H < H_c$ (H is the effective field) and

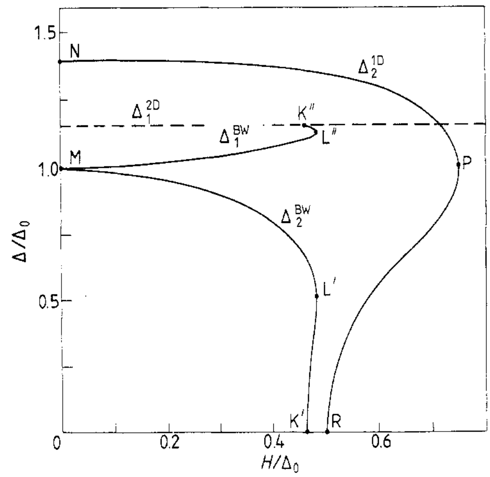

Figure 4. $\Delta(H)$ functions for BW anisotropic, polar 1D and planar 2D states ($T = 0$).

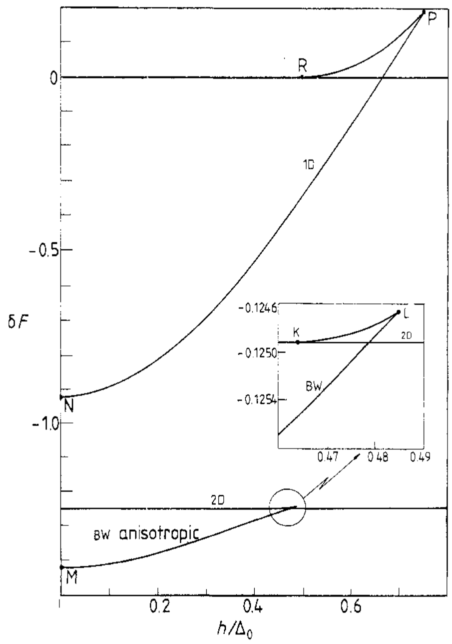

Figure 5. Free energies δF as functions of H for BW anisotropic, 1D and 2D states ($T = 0$).

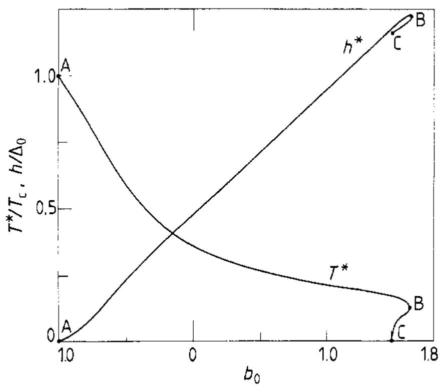

Figure 6. Coordinates of the tricritical points for the BW–2D transition.

the 2D state for $H > H_c$ (cf. figures 4 and 5). This stabilisation of the 2D state in the vicinity of T_c has been described by Hasegawa (1980), Tewordt and Schopohl (1979), Jacak (1981), and figures 4 and 5 are similar to those in Schopohl (1979); the same is so in the case of figures 1 and 3(c). The 2D state is field-independent in the framework of the weak coupling theory. Moreover, other states with $m = 1$ or $m = 2$ are unstable within the weak coupling approach (similar zero-field limit). Therefore, only the phase transition BW (anisotropic)–2D is of physical interest (in the weak coupling theory). As in the case of S-SFL, here we also deal with the paramagnetic instability phenomenon for this transition. By solving the appropriate bifurcation type condition, in analogy with the consideration in § 2, we can determine the tricritical point coordinates on the BW–2D transition curve. These coordinates are plotted against b_0 in figure 6.

In summary, we can state that for $-1 < b_0 < 1.51$ a single tricritical point exists, while for $1.51 \leqslant b_0 < 1.65$ two tricritical points exist. For $b_0 > 1.65$ the paramagnetic instability phenomenon disappears entirely. Application of the above analysis to the superfluid ^{3}He, for which $b_0 \simeq -0.7$, shows that its behaviour corresponds to the first region of b_0, so that paramagnetic instability should occur for ^{3}He-B with a tricritical point at $T^* = 0.814T_c$, $h^* = 0.078\Delta_0$. Nevertheless it is well known that the strong coupling corrections qualitatively change the He_3 phase diagram and would disturb this paramagnetic instability for BW–2D by removing the degeneration of two-dimensional states and by stabilising the ABM phase (cf. Leggett 1975).

4. The inhomogeneous LO-type state for S-SFLS

It is known that in the neutral BCS gas in the presence of a magnetic field, the inhomogeneous superfluid state appears to be more stable (at a particular magnetic field value) than the usual BCS state (Larkin and Ovchinnikov 1964, Fulde and Ferrell 1964). In the case of SFLS, however, the stability of the LO state is also very sensitive to the magnitude of the spin-exchange quasi-particle interaction. In order to demonstrate this let us collect a self-consistent system of equations for the anomalous and normal parts of the mass operator, i.e., for the gap and renormalised magnetic field, supplemented with the Bloch condition which guarantees that the total current will vanish (cf Takada and Izuyama 1969). These equations have the form

$$\Delta_q = g_0 \Delta_q \sum_p \frac{1}{E_p} [1 - f(E_p + W_p) - f(E_p - W_p)] \tag{16}$$

$$\boldsymbol{\Sigma} = \sum_{l=0,2,\ldots} \frac{2l+1}{N_0} b_l \sum_{p'} p_l(\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{p}}') \hat{H} [f(E_{p'} + W_{p'}) - f(E_{p'} - W_{p'})] \tag{17}$$

$$\begin{aligned} \sum_p \left(p + \frac{q}{2}\right) \{[f(E_p + H + z) + f(E_p - H + z)](1 + \xi_p/E_p) \\ + [f(-E_p - H + z) + f(-E_p - H + z)](1 - \xi_p/E_p)\} = 0 \end{aligned} \tag{18}$$

where Δ_q denotes the amplitude of the gap, the spatial inhomogeneity of which is determined by the $\boldsymbol{q}$ momentum vector. Moreover

$$E_p = (\xi_p^2 + \Delta_q^2)^{1/2} \qquad W_p = H + z \qquad z = (\alpha/2)\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}} \qquad \alpha = q v_F.$$

Let us now consider the $T = 0$ limit, when the integrals in equations (16)–(18) can be solved explicitly. In this limit the system (16), (18) takes the form:

$$\begin{aligned} \Delta_q \ln \frac{\Delta_q}{\Delta_0} = \frac{\Delta_q}{\alpha} \Big[\Theta(C_- - \Delta_q) \left(C_- \ln \frac{C_- + D_-}{\Delta_q} - D_- \right) + \Theta(-C_- - \Delta_q) \\ \times \left(C_- \ln \frac{C_- + D_-}{\Delta_q} + D_- \right) + \Theta(C_+ - \Delta_q) \left(-C_+ \ln \frac{C_+ + D_+}{\Delta_q} + D_+ \right) \Big] \end{aligned} \tag{19}$$

$$\Sigma = -\frac{b_0}{\alpha} \left[\Theta(C_- - \Delta_q) \left(-\frac{C_- D_-}{2} + \frac{\Delta_q^2}{2} \ln \frac{C_- + D_-}{\Delta_q} \right) \right.$$

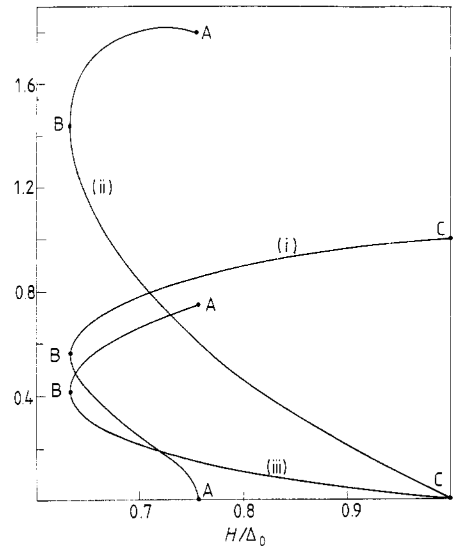

Figure 7. Solutions to equations (19, 21) for (i) $\Delta_q(H)$, (ii) $\alpha(H)$ and (iii) $\Sigma/b_0(H)$. A, B, C denote corresponding branches of the solutions. The coordinates $H/\Delta_0, \Delta_q/\Delta_0, \alpha/\Delta_0, \Sigma/b_0/\Delta_0$ are 0.754, 0.0, 1.810, 0.754, 0.634, 0.556, 1.426, 0.412 and 1.0, 1.0, 0.0, 0.0 for A, B, C, respectively ($T = 0$).

$$+\,\Theta(-C_- - \Delta_q)\left(\frac{C_-D_-}{2} + \frac{\Delta_q^2}{2}\ln\frac{-C_- + D_-}{\Delta_q}\right)$$

$$+\,\Theta(C_+ - \Delta_q)\left(\frac{C_+D_+}{2} - \frac{\Delta_q^2}{2}\ln\frac{C_+ + D_+}{\Delta_q}\right)\Bigg] \tag{20}$$

$$0 = \frac{\alpha}{6} + \frac{1}{\alpha^2}\Bigg[\Theta(C_- - \Delta_q)\left(\tfrac{2}{3}D_-^3 - C_-D_-H + H\Delta_q^2\ln\frac{C_- + D_-}{\Delta_q}\right)$$

$$+\,\Theta(-C_- - \Delta_q)\left(-\tfrac{2}{3}D_-^3 + C_-D_-H + H\Delta_q^2\ln\frac{-C_- + D_-}{\Delta_q}\right)$$

$$+\,\Theta(C_+ - \Delta_q)\left(-\tfrac{2}{3}D_-^3 + C_+D_+H - H\Delta_q^2\ln\frac{C_+ + D_+}{\Delta_q}\right)\Bigg] \tag{21}$$

where $C_\pm = H \pm \alpha/2$, $D_\pm = (C_\pm^2 - \Delta_q^2)^{1/2}$ and $\Theta(x)$ is Heaviside step function. The step functions in equations (19)–(21) describe, in the formal manner, the 'blocking' effects essential for the LO state, and that exhibit the interplay between the depairing action of the magnetic field and superflow.

Note that b_0 explicitly enters equation (20), while equations (19) and (21) can be solved in terms of H. This allows the functions $\Delta_q = \Delta_q(H)$, $\alpha = \alpha(H)$ and $\Sigma/b_0 = \Sigma/b_0(H)$ to be determined numerically ($-N_0\Sigma/b_0$ is the spin magnetisation of the system), and are plotted in figure 7. Note also that the critical magnetic field H_c and critical value of α, α_c (i.e., the coordinates of the point A in figure 7) can be found from equations (19) and (21), both taken in the limit $\Delta_q \to 0$, i.e.,

$$1 - \ln 2\Delta_0 + \frac{H_c}{\alpha_c}\ln\left|\frac{H_c - \alpha_c/2}{H_c + \alpha_c/2}\right| - \tfrac{1}{2}\ln\left|\frac{H_c^2 - \alpha_c^2/4}{\Delta_0^2}\right| = 0 \tag{22}$$

$$1 = (H_c/\alpha_c)\ln\left|\frac{H_c - \alpha_c/2}{H_c + \alpha_c/2}\right|. \tag{23}$$

From these equations one can find $H_c = 0.754\Delta_0$ and $\alpha_c = 1.810\Delta_0$, where Δ_0 is the BCS gap for $T = h = 0$. The position of point C in figure 7 is determined by the appropriate BCS-type equation, i.e., it corresponds to the $\alpha \to 0$ limit. The coordinates of point B can be found by employing the condition $\partial h/\partial \Delta_q = 0$.

The curves plotted in figure 7 constitute a complete (at $T = 0$) LO-type solution for $b_0 = 0$ (since in this case $H = h$). For arbitrary $b_0 \neq 0$, these curves could be easily transformed into the appropriate $\Delta_q = \Delta_q(h)$ and $\alpha = \alpha(h)$ via renormalisation of the abscissa according to $H = h + \Sigma$.

The crucial point in the description of this system lies in the evaluation of the free energies of normal, BCS and LO states at the same h, since it will reflect the energetical competition between these states. In order to calculate the free energy of inhomogeneous SFLs one can use the integration of the self-consistent equation with respect to the interaction constants. In the limit $T = 0$ the appropriate formula takes the form

$$\begin{aligned}\delta F = F_S^{(LO)}(h, \alpha) - F_N(h) = (\lambda/2)\{&-\Delta_q^2/2 + H^2 + \alpha^2/12 \\ &+ (1/3\alpha)[\Theta(C_- - \Delta_q)D_-^3 - \Theta(-C_- - \Delta_q)D_-^3 - \Theta(C_+ - \Delta_q)D_+^3] \\ &- [b_0/(1 + b_0)][h + \Sigma(1 + b_0)/b_0]^2\}\end{aligned} \tag{24}$$

where $F_S^{(LO)}$, F_N are the free energies of superfluid LO and normal states in the presence of a magnetic field, respectively. Let us define

$$\delta F_o = \delta F + [b_0/(1 + b_0)][h + \Sigma(1 + b_0)/b_0]^2\, \lambda/2.$$

This function in the stationary points (i.e., following the solutions from figure 7) is plotted in figure 11. Note that $\delta F(h, b_0 = 0) = \delta F_0(H)$. To evaluate the free energy $\delta F(h)$ (for $b_0 \neq 0$) one can use the numerical results for $\delta F_0(H)$ and $\Sigma/b_0(H)$, and the condition $H = h + \Sigma$.

Due to equation (24) one can ensure (taking the limit $\Delta_q \to 0$) that the LO state (with $\alpha \neq 0$) is energetically more favourable than the normal state provided that $b_0 > -0.774$ only. The explicit form of the condition for this limiting value of b_0 is as follows:

$$\frac{b_0}{1 + b_0} \ln^2 \left| \frac{H_c - \alpha_c/2}{H_c + \alpha_c/2} \right| = \frac{3\alpha_c^2 \Delta_0^2}{2(H_c^2 - \alpha_c^2/4)}. \tag{25}$$

On the other hand, it is necessary to compare the free energy of the LO state with that of the BCS state. The free energy of the latter is given by formula (10) (with equations (6) and (7)). It can be established that there are two regions of b_0, with the distinct behaviour of the system. To be specific, for $-1 < b_0 < -0.122$ the stable superfluid state is of the BCS type and exists for $0 \leqslant h < \Delta_0[(1 + b_0)/2]^{1/2}$. In this case the LO state does not minimise the free energy at all. For the region $-0.122 < b_0 \leqslant 3.40$ we observe the stable BCS state for $0 \leqslant h < h_{LO}$, and for $h_{LO} < h < H_c(1 + b_0)$ the stable state is of LO type (for $h > H_c(1 + b_0)$ stable is the normal state). The phase transition BCS–LO (at $h = h_{LO}$) is here first order, while the transition LO–normal is second order. However, if b_0 exceeds 3.40, the jump in the gap (and magnetisation) at $h = h_{LO}$ becomes less than $10^{-4}\,\Delta_0$. Thus from the physical point of view it is irrelevant to distinguish between first- and second-order phase transitions at this point. The jump in the gap $\delta\Delta = \Delta_0 - \Delta_q(h = h_{LO})$ and in the appropriate α $\delta\alpha = -\alpha(h = h_{LO})$ are plotted in figure 8, and the function $h_{LO}(b_0)$ is plotted in figure 9.

For $b_0 = \frac{1}{3}$ (which value is assumed for Al), the solution $\Delta_q(h)$ is presented in figure 10, and the appropriate free energy is plotted in figure 11. Note that the Landau

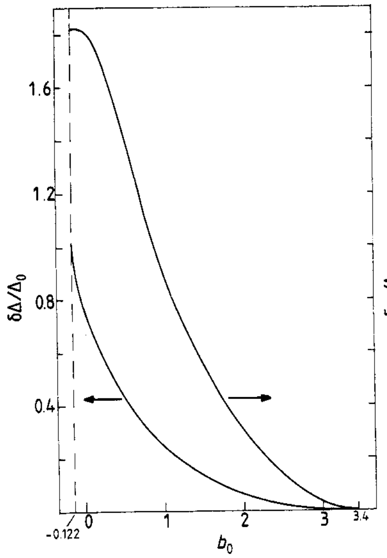

Figure 8. The jump in the gap $\delta\Delta = \Delta_0 - \Delta_q(h = h_{LO})$ and the jump in α parameter $\delta\alpha = -\alpha\ (h = h_{LO})$ at the first-order BCS–LO phase transition.

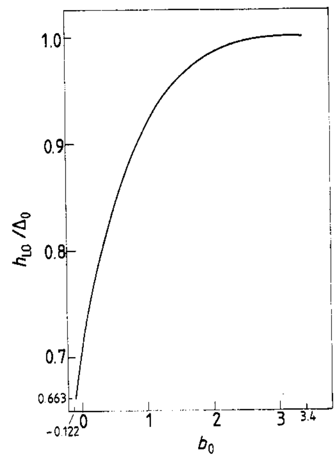

Figure 9. Critical magnetic field for the BCS–LO phase transition h_{LO} versus b_0.

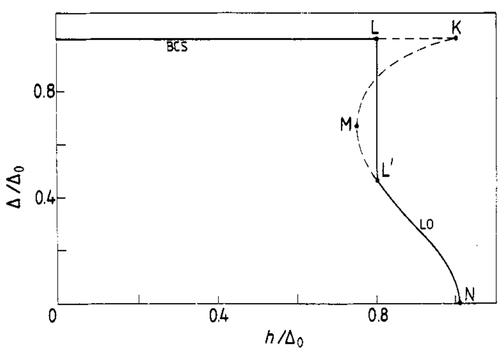

Figure 10. Energy gap as a function of h for $b_0 = \frac{1}{3}$.

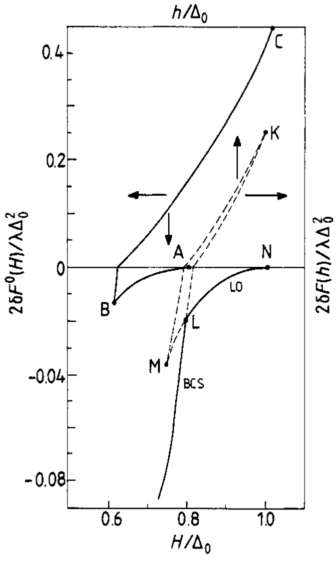

Figure 11. Free energy $\delta F = F_S - F_N$ as a function of h for $b_0 = \frac{1}{3}$; the free energy δF_0 at its stationary points as a function of H.

parameter b_0 influences not only the stability range of the LO state, but also changes (enhances for $b_0 > 0$) the energetical distance between LO and BCS (or normal) states (cf. figures 9 and 11). In this context the inclusion of impurity scattering (Takada 1970, Aslamazov 1969) always diminishes the stability of the LO state.

5. Orbital depairing action of the magnetic field for s- and p-SFL

The orbital depairing action of the magnetic field cannot be generally discussed without considerable modification of the superfluid state order parameter, i.e., one has to consider the inhomogeneous gap parameter. Although the general Gorkov equations can be written in the position space for the inhomogeneous case the solutions are known only in the framework of linear approximation (this also holds for the phenomenological Landau–Ginzburg-type approach). These problems are due to the inconvenience of position representation. Nevertheless, in the local limit, when the superfluid velocity is assumed to be spatially uniform, the usual momentum representation can be applied with a diagonal one-particle Green function. This limit, also known as the London limit, is a realistic approximation for superconductors with magnetic field penetration depths exceeding the coherence length. Thus it suffices to describe the Meissner phase (and even mixed state, but not close to the H_{c_2} field) of type II superconductors; cf. Svidzinskii (1982). This circumstance makes the local limit interesting from physical point of view, and its inherent simplicity allows efficient discussion of Fermi liquid corrections. Let us first consider the singlet s-paired isotropic Fermi liquid. The self-consistent system of equations which substitute the usual gap equation takes the form:

$$\Delta = \frac{\lambda}{2}\Delta \int_0^{\omega_D} \mathrm{d}\xi \int_0^1 \mathrm{d}x \left(\tanh\frac{E_+}{2T} + \tanh\frac{E_-}{2T}\right)\frac{1}{E} \tag{26}$$

$$\Sigma = -\tfrac{3}{2}a_1 \int_0^{\infty} \mathrm{d}\xi \int_0^1 \mathrm{d}x\, x \left(\tanh\frac{E_+}{2T} - \tanh\frac{E_-}{2T}\right) \tag{27}$$

where $E_\pm = E \pm Ax$, $A = vp_F + \Sigma$, $x = \hat{p}\cdot\hat{v}$, v is the superfluid velocity and a_1 is the first spinless Landau amplitude, the same as that involving the effective mass formula. Note that only this amplitude appears to be important with respect to the velocity field in the isotropic system, provided that the spontaneous breakdown in rotational symmetry is not considered. These non-linear equations cannot be solved analytically in the general case. Nevertheless, in the $T = 0$ limit both integrals in equations (26) and (27) can be solved, and we then obtain a system of algebraic equations:

$$\begin{aligned}\Delta = \lambda\Delta\{&\ln 2\omega_D - \Theta(\Delta - A)\ln\Delta + \Theta(A - \Delta)(A^2 - \Delta^2)^{1/2}/A \\ &- \Theta(A - \Delta)\ln[(A^2 - \Delta^2)^{1/2} + A]\}\end{aligned} \tag{28}$$

$$\Sigma = -a_1\Theta(A - \Delta)(A^2 - \Delta^2)^{3/2}/A^2 \tag{29}$$

with $1/\lambda = \ln(2\omega_D/\Delta_0)$, $\Delta_0 = \Delta(T = 0, v = 0)$. Even though this system also turns out to be unmanageable, some characteristic points can be determined analytically:

(i) if $A \to 0$ (or even $\Delta > A$ only) then $\Delta = \Delta_0$;

(ii) if $\Delta \to 0$ then $A = e\Delta_0/2$ and, on the other hand, $A \to A_N = vp_F/(1 + a_1)$; thus $v_c = (1 + a_1)e\Delta_0/2p_F$ is the velocity at which the gap Δ tends continuously to zero;

(iii) $\Delta = \Delta_0$ and $A = \Delta_0$.

Numerical methods have been employed to solve equations (28)–(29). The solutions $\Delta(v)$ for several values of a_1 are plotted in figure 12. The curve $\Delta(v)$ for $a_1 = 0$ coincides with the solution $\Delta(A)$ of equation (28), which is independent of a_1. This solution was further transformed by renormalisation of abscissa according to equation (29) to obtain the various Landau parameter curves. It is important to note that at $a_1 = -\frac{1}{7}$ the topology

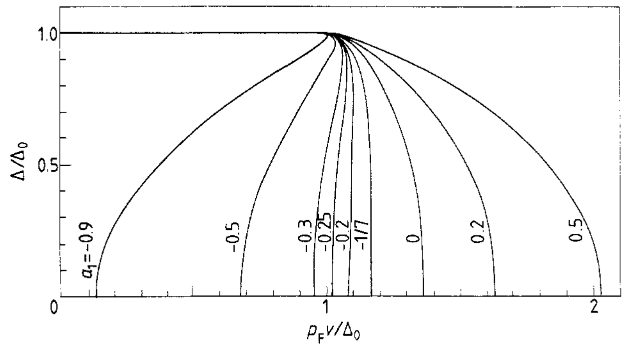

Figure 12. Gap Δ with respect to v for several values of the Landau parameter a_1 ($T=0$).

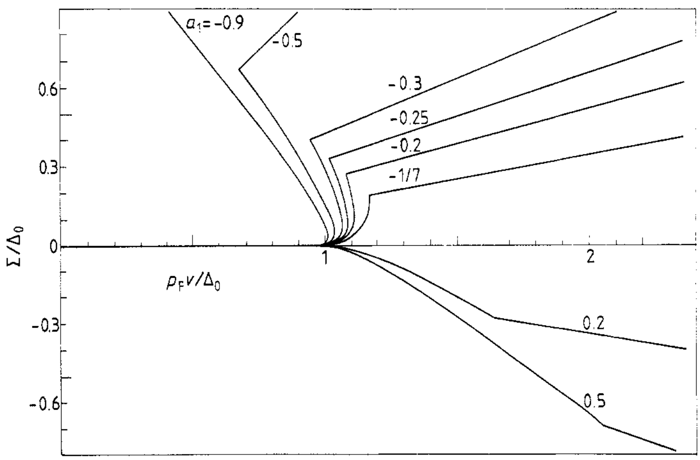

Figure 13. Mass operator Σ with respect to v for several values of the Landau parameter a_1 ($T=0$).

of $\Delta(v)$ solutions changes. For $a_1 > -\frac{1}{7}$ we deal with single-value non-zero functions $\Delta(v)$, $\Sigma(v)$, while for $-1 < a_1 < -\frac{1}{7}$ these functions are double-valued in certain parts of the area of determination (see also figure 13). The critical value $a_1 = -\frac{1}{7}$ has been determined by applying the bifurcation-type condition to the non-linear system of equations (28) and (29) for $T=0$. More generally, at arbitrary temperatures the bifurcation condition assumes the form (in the limit $\Delta \to 0$):

$$\left[\int_0^\infty d\eta \left(\frac{1}{\eta^2}[\tanh(\eta+\alpha) - \tanh(\eta-\alpha)] - \frac{1}{\eta^3}\ln\frac{\cosh(\eta+\alpha)}{\cosh(\eta-\alpha)}\right)\right](1+a_1) - \tfrac{3}{2}a_1\alpha$$
$$\times\left(\int_0^\infty d\eta \int_0^1 dx\,[\cosh^{-2}(\eta+\alpha x) - \cosh^{-2}(\eta-\alpha x)]\frac{x}{\eta}\right)^2 = 0 \tag{30}$$

$$\int_0^\infty \frac{d\xi}{\xi}\left(\frac{T}{A}\ln\frac{\cosh(\xi+A)/2T}{\cosh(\xi-A)/2T} - \tanh\frac{\xi}{2T_c}\right) = 0 \tag{31}$$

$$\Sigma = -\,a_1 v p_F/(1+a_1) \qquad \text{where } \alpha = A/2T. \tag{32}$$

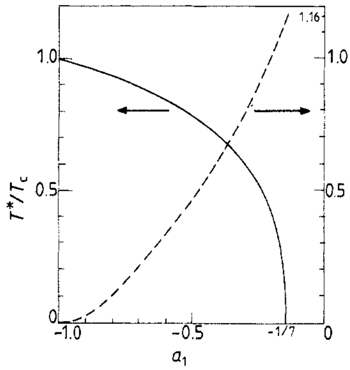

Figure 14. Coordinates of the tricritical point on the T, v plane with respect to the Landau parameter a_1.

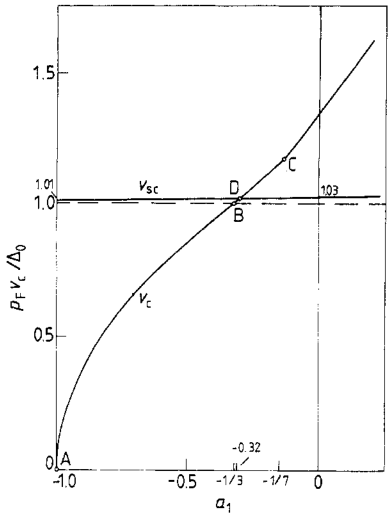

Figure 15. Critical values of v with respect to a_1 (the superfluid–normal phase transition) and v_{sc} with respect to a_1.

The solution of the above system enables determination of the tricritical point coordinates versus a_1, and the appropriate curves are plotted in figure 14. At this tricritical point the superfluid–normal phase transition changes order; first order for $0 \leq T < T^*$ and second order for $T^* < T \leq T_c$.

For the first-order phase transition case, the free energy needs to be evaluated to determine the critical superfluid velocity. According to the Feynman method the appropriate free energy takes the form:

$$F_{S,v} - F_{N,v} = -\frac{\lambda}{4}\int_0^{\Delta} d\Delta'\,\Delta'^3 \int_0^{\infty} d\xi \int_0^1 dx \left[\frac{1}{E^3}\left(\tanh\frac{E_+}{2T} + \tanh\frac{E_-}{2T}\right) - \frac{1}{2TE^2}\right.$$
$$\left.\times\left(\frac{1}{\cosh^2(E_+/2T)} + \frac{1}{\cosh^2(E_-/2T)}\right)\right] - \frac{\lambda}{6}\frac{a_1}{1+a_1}\left(vp_F + \Sigma\frac{1+a_1}{a_1}\right)^2 \tag{33}$$

where $F_{S,v}$ and $F_{N,v}$ are free energies of the superfluid and normal phases, respectively (both for non-zero superfluid velocity). At $T = 0$ the above expression resolves itself to the formula:

$$F_{S,v} - F_{N,v} = \frac{\lambda}{2}\left[-\frac{\Delta^2}{2} + \frac{A^2}{3} - \Theta(A-\Delta)\frac{(A^2-\Delta^2)^{3/2}}{3A} - \frac{1}{3}\frac{a_1}{1+a_1}\right.$$
$$\left.\times\left(vp_F + \Sigma\frac{1+a_1}{a_1}\right)^2\right]. \tag{34}$$

The resulting critical velocity versus a_1 is presented in figure 15 (for $T = 0$). The part AB (i.e., for $-1 < a_1 < -\frac{1}{3}$) of the $v_c(a_1)$ is determined by $v_c = [\frac{3}{2}(1 + a_1)]^{1/2}\,\Delta_0/p_F$. The part BC (for $-\frac{1}{3} < a_1 < -\frac{1}{7}$) is not governed by any analytically given dependence (it is found numerically). The third part (for $a_1 > -\frac{1}{7}$) corresponds to the dependence $v_c = (e/2p_F)(1 + a_1)$, and is related to the second-order phase transition.

Let us now calculate the current in the system considered:

$$j_s = \frac{2T}{a}\sum_{p,\nu}\frac{p + mv}{m} G(i\omega_\nu, p) = \frac{\rho_s}{m} v = \frac{\rho}{mv} - j_n \tag{35}$$

where G is the Matsubara Green function,

$$j_n = -\frac{2T}{a}\sum_{p,\nu}\frac{p_F}{m}\hat{p}G(i\omega_\nu, p) = \frac{\rho_n}{m} v$$

and

$$\rho = \frac{2T}{a}\sum_{p,\nu} G(i\omega_\nu, p) = \frac{N_0 p_F^2}{3(1 + a_1)}$$

is the density. These expressions form the usual hydrodynamic description of the two-component system, where ρ_n, ρ_s correspond to densities of normal and superfluid components, respectively. On the other hand, one can note that

$$j_n = -(\rho/mp_F)[(1 + a_1)/a_1]\Sigma\hat{v}. \tag{36}$$

Hence the determination of Σ yields the evaluation of ρ_n as well as ρ_s, i.e.,

$$\rho_s = \rho\{1 + [(1 + a_1)/a_1]\Sigma/vp_F\}. \tag{37}$$

Typical shapes of Σ are plotted in figure 13. Note that according to the definition of A, $A = vp_F + \Sigma$, one can rewrite equation (35) as

$$j_s = \frac{\rho}{mp_F}\left(vp_F + \frac{1 + a_1}{a_1}\Sigma\right)\hat{v} = \frac{\rho}{mp_F}\left(A + \frac{\Sigma}{a_1}(A)\right)\hat{v}. \tag{38}$$

It is important to observe that the current j_s (i.e., superflow in the laboratory coordinate system) depends on a_1 via A only, since Σ/a_1 is a function of A. Bearing in mind that the critical depairing superflow is defined as the maximal value of j_s (which is obtained at A_{sc}, according to $dj_s/dA = 0$) one can state that the critical depairing superflow is not affected by a_1, unless $v_{sc} > v_c$, where v_c is the critical velocity (at which the transition to normal phase takes place) and v_{sc} is the bare velocity corresponding to A_{sc} (suitably to a_1). This last correspondence is governed by

$$v_{sc}p_F = A_{sc} - (\Sigma/a_1)(A_{sc})a_1. \tag{39}$$

Thus the function $v_{sc}(a_1)$ turns out to be linear. To be specific, let us now consider the $T = 0$ limit, when equation (38) attains the form:

$$j_s = (\rho/mp_F)\{A - \Theta(A - \Delta)[(A^2 - \Delta^2)^{3/2}/A^2]\}. \tag{40}$$

This function is plotted in figure 16, and $A_{sc} = 1.03\Delta_0$, $j_{sc} = 1.01(\rho\Delta_0/mp_F)$. Hence at $T = 0$ equation (39) has the form

$$v_{sc}p_F/\Delta_0 = 1.03\text{–}0.02a_1$$

as shown in figure 15. From this figure one can notice that the line $v_{sc}(a_1)$ intersects the curve $v_c(a_1)$ at point D (i.e., $a_1 = -0.32$). Thus for $-1 < a_1 < -0.32$ we have $v_c < v_{sc}$ and the determined j_{sc} cannot be attained, since the transition to normal phase (of first

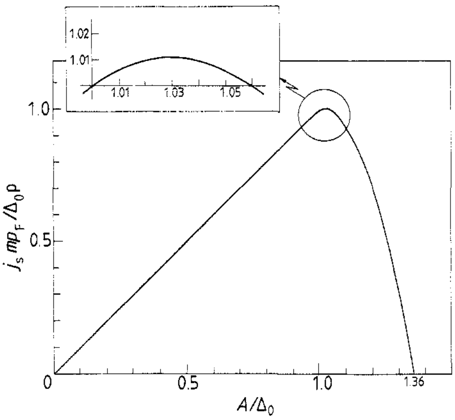

Figure 16. Superflow j_S with respect to A ($T = 0$).

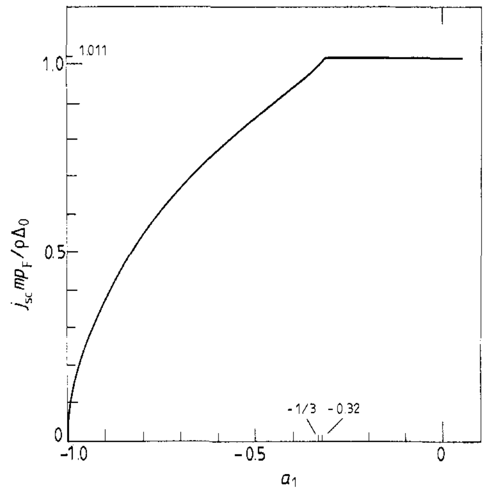

Figure 17. Critical depairing superflow j_{SC} with respect to a_1.

order) takes place earlier. The resulting critical depairing superflow is plotted against a_1 in figure 17.

The analogous consideration can be carried out for the other kind of pairing. Of special interest is to present the appropriate results for p-SFL, since the local limit for orbital magnetic field action coincides with superflow action for the neutral system and thus suits the description of superfluid ^{3}He. Let us consider the gap parameter in the form given by equation (12) where $\hat{q}$ indicates the unit vector in the direction of superfluid velocity $\boldsymbol{v}$. The gap and the mass operator equations have the form (Vollhardt *et al* 1980):

$$\Delta_1 = \Delta_1 \tfrac{3}{4}\lambda \int_0^{\omega} \mathrm{d}\xi \int_0^1 \mathrm{d}x\,(1-x^2)\,\frac{1}{E}\left(\tanh\frac{E_+}{2T} + \tanh\frac{E_-}{2T}\right) \tag{41}$$

$$\Delta_2 = \Delta_2 \tfrac{3}{4}\lambda \int_0^{\omega} \mathrm{d}\xi \int_0^1 \mathrm{d}x\,2x^2\,\frac{1}{E}\left(\tanh\frac{E_+}{2T} + \tanh\frac{E_-}{2T}\right) \tag{42}$$

$$\Sigma = -3a_1 \int_0^{\infty} \mathrm{d}\xi \int_0^1 \mathrm{d}x\,\frac{x}{2}\left(\tanh\frac{E_+}{2T} - \tanh\frac{E_-}{2T}\right) \tag{43}$$

where $E_\pm = E \pm Ax$, $A = vp_F + \Sigma$ and $E = (\xi^2 + \Delta_1^2(1-x^2) + \Delta_2^2 x^2)^{1/2}$. This system cannot be solved analytically. Let us now discuss its solution at $T = 0$ K. In this limit the equation system is also unmanageable, although both integrations with respect to ξ and x can be explicitly performed. Similar to the Pauli depairing action, we find here also the phase transition between the BW anisotropic state (with $\Delta_1 \neq 0$, $\Delta_2 \neq 0$) and the 2D planar state (with $\Delta_1 \neq 0$, $\Delta_2 = 0$). In figures 18 and 19 the gaps Δ_1, Δ_2 and mass operator Σ are plotted as functions of 'dressed' superfluid velocity $A = vp_F + \Sigma$, at $T = 0$. The distinct behaviour here, in comparison with the Pauli depairing picture, (cf. figure 4) is remarkable in that (1) the 2D gap parameter is dependent on the superfluid velocity, and (2) the shapes of the $\Delta_1(A)$ and $\Delta_2(A)$ curves are distinct compared with $\Delta_1(H)$ and $\Delta_2(H)$. We have examined the phase transitions BW–2D and 2D–normal with respect to their order and determined that $a_1 = 0.966$ is the position of the tricritical point (at $T =$

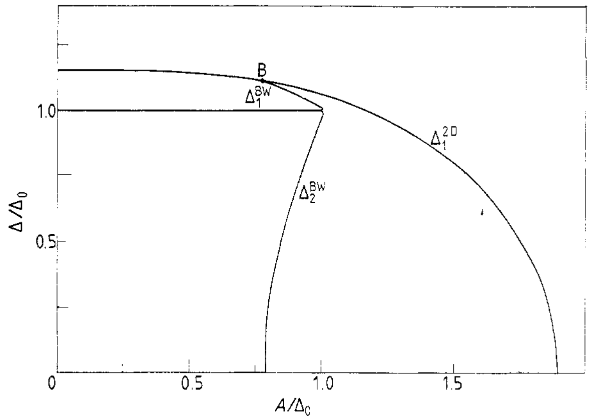

Figure 18. Order parameters Δ_1 and Δ_2 for BW anisotropic and 2D states with respect to A ($T = 0$).

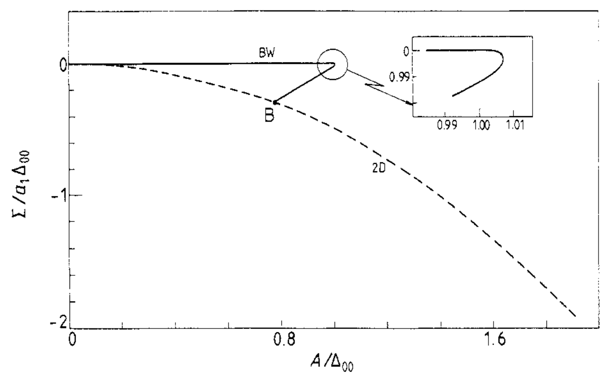

Figure 19. Mass operators for BW anisotropic and 2D states with respect to A ($T = 0$).

0) for the phase transition BW–2D. For $-1 < a_1 < 0.966$ this transition is first order, while for $a_1 > 0.966$ it is second order. For the 2D–normal phase transition the tricritical point is at $T = 0$ for $a_1 = -\frac{1}{2}$ (for $-1 < a_1 < -\frac{1}{2}$ the phase transition is first order and for $a_1 > -\frac{1}{2}$ second order).

As in the case of s-SFL, the first-order phase transition to 2D or normal states can hamper the depairing superflow to attain its maximal value for BW and 2D states, respectively. The critical superflows for 2D and BW anisotropic states are plotted in figure 20 as functions of the Landau parameter a_1 (at $T = 0$).

Although the results obtained contradict the statement (Vollhardt *et al* 1980) that the critical superflow is not at all affected by the Landau interaction, in the case of ^{3}He the dependence presented above turns out to be insignificant since for ^{3}He, $a_1 = 2.01$ (at zero pressure) and $a_1 = 5.22$ (at melting pressure), i.e., they are in the region of values within which the dependence $j_{sc}(a_1)$ disappears.

6. Final remarks

As has already been mentioned, the paramagnetic critical phenomena described in §§ 2–4 cannot be applied to a description of real metallic superconductors, since the orbital

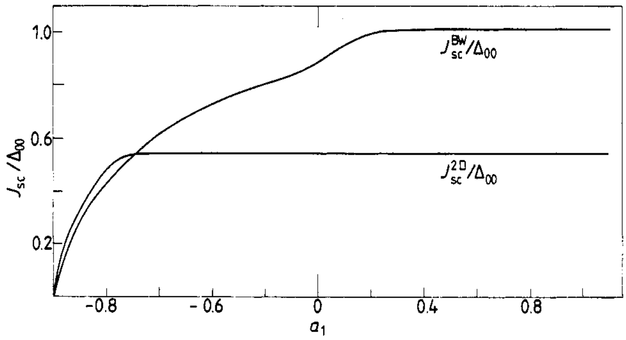

Figure 20. Critical depairing superflows for BW anisotropic and 2D states ($T = 0$).

depairing action of the magnetic field usually exceeds the action of the Pauli term (in the framework of the London limit at least). The Pauli limiting field in the calculation of the upper critical field for type II superconductors is partially justified, however, since the local limit is not a good approach in the mixed phase and the coherence length is no better for the spatial scale. Moreover, in some special materials one can expect an enlargement of the effective Bohr constant due to the exchange interaction of electron spin with the admixture spins. Such a situation is believed to be realised in the ferromagnetic rare earth ternary compound $ErRh_4B_4$. Fujita *et al* (1984) have applied the paramagnetic theory of BCS gas to the 4f moment system and fitted the $H_{c_2}(T)$ curve along the *a* axis for $ErRh_4B_4$ (experimental curve after Grabtree *et al* 1982). The BCS gas model leads, however, to a discrepancy between theoretical and experimental values of T^*, i.e., $(T^*/T_c)_{theor} = 0.556$ while $(T^*/T_c)_{exp} = 0.425$.

In order to remove this discrepancy let us include the quasi-particle interaction in the SFL theoretical framework lifted suitably to include the s–f exchange interaction. The appropriate procedure (similar to that of Fujita *et al* 1984) resolves itself to an additional renormalisation of the effective magnetic field

$$H = h_{ex}\alpha(T) + \Sigma \tag{44}$$

where

$$\alpha(T) = 1 + [4\pi + I(g_J - 1)/Ng_J\mu_B^2]\chi_f(T).$$

Here I is the s–f interaction constant, g_J is the Landé g-factor, N is the number of magnetic ions per unit volume, h_{ex} is the external DC magnetic field (in direction of the a-axis for $ErRh_4B_4$). The magnetic susceptibility of a magnetically polarisable medium of a 4f local-moment system is given by the Curie–Weiss formula:

$$\chi_f(T) = N(g_J\mu_B)^2 J(J+1)/3(T - T_M) \tag{45}$$

where T_M is the ferromagnetic transition temperature and J is the total angular momentum. This magnetic field renormalisation yields an overall bell-shaped critical field curve obtained via equations (1) and (2) (with H given by equation (44); cf. figure 21). To fit the position of the cross-over point on the H_{c_2} curve (point P in figure 21) b_0 is chosen to be 0.24 since, for this value of interaction constant, T^* coincides with its experimental magnitude (cf. figure 2). From the curve $h^*(b_0)$ (cf. figure 2) it is possible to determine a suitable H^* and further via equations (44) and (45) one can find $I = 70$ K provided g_J, J and N hold the values: $g_J = 1.2, J = 7.5, N = 9.62 \times 10^{12}$ atoms cm^{-3} (for $I = 70$ K, h_{ex}

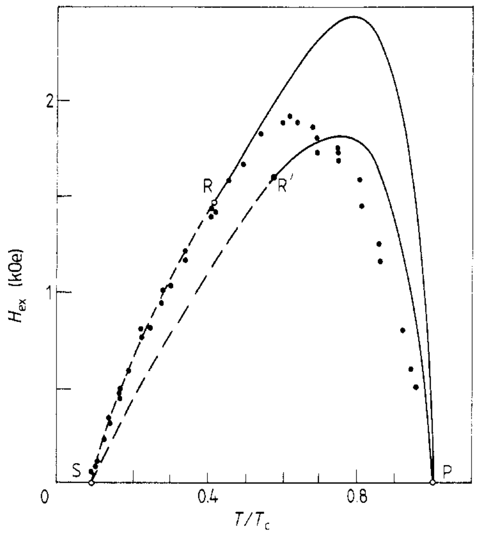

Figure 21. The upper critical field $H_{c_2}(T)$ for $ErRh_4B_4$, a-axis. Dots represent experimental data after Grabtree *et al* (1982). Full curve PR (broken curve RS) corresponds to the second- (first)-order phase transition. The line (PR′S) corresponds to H_{c_2} after Fujita *et al* (1984).

coincides with 1.44 kOe). The curve in figure 21 is plotted for these parameters. The full curve PR and broken curve RS in figure 21 correspond to the second- and first-order phase transitions, respectively; the point R is the tricritical cross-over point. The curve PRS, even though it possesses the typical bell shape and fits with experimental data in the first-order phase transition region, does not coincide with experimental curve in the second-order transition region. The inclusion of a finite electronic mean path via one more parameter $1/\tau_{so}$ (arising from spin–orbit interactions) can diminish this discrepancy, since $1/\tau_{so}$ changes the critical field in the distinct manner of Landau interactions (Bruno and Schwartz 1973). Note, however, that inclusion of spin–orbit scattering decreases the value of T^* (Bruno and Schwartz 1973), so in this case b_0 should be taken to be smaller than 0.24; this value can be treated as the upper limit for spin-exchange interaction constant for $ErRh_4B_4$.

With regard to other applications of the above consideration, neutral s-SFL is an ideal model with which to describe the hypothetical superfluidity of ^3He in ^3He–^4He when the bosonic medium makes singlet pairing more convenient than the triplet one. The experimental possibility of detecting the appropriate transition is nevertheless complicated by its extremely low critical temperature (theoretically estimated to vary between 10^{-4} K and 10^{-8} K; cf. Ivanova and Mejerovich 1986). Moreover, for a dilute mixture of ^3He (its limiting concentration at $T = 0$ is approximately 6.5%), b_0 is given by

$$b_0 \simeq (2\pi a\hbar^2)/m^* \tag{46}$$

where a (the mean free path of an ^3He atom) is of the order of 0.5–1.5 Å at 3% concentration, and m^* is the effective mass of a ^3He quasi-particle. Thus b_0 is small and positive, which should make the paramagnetic behaviour of ^3He–^4He mixture differ very little from that of a BCS gas.

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